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# CONTRIBUTION OF THE EXCHANGE MECHANISM TO THE BACKWARD SCATTERING OF PROTONS BY LIGHT NUCLEI AND THE d + $^{4}He \rightarrow {}^{3}He + {}^{3}H$ REACTION

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# CONTRIBUTION OF THE EXCHANGE MECHANISM TO THE BACKWARD SCATTERING OF PROTONS BY LIGHT NUCLEI AND THE $d + {}^4He \to {}^3He + {}^3H$ REACTION

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ABSTRACT. The role of the exchange mechanism in backward scattering of protons by ${}^3He$ and ${}^4He$ nuclei is discussed. The simplest exchange diagram for backward scattering of protons by ${}^4He$ is calculated by means of the wave function, the parameters of which were fixed by data concerning the scattering of electrons. A good agreement with the experimental data was attained. In the energy dependence of the backward-scattering cross-section of protons with ${}^4He$ nuclei, a characteristic dip in the region of 200 Mev is anticipated. An evaluation was performed of the parameters which determine the behavior of the wave function of ${}^3He$ at short distances. The $d + {}^4He \to {}^3He + {}^3H$ reaction was examined. Experiments were proposed for clarifying the mechanism responsible for the processes examined.

## 1. Introduction /3*

There has recently been a considerable enrichment of experimental data concerning elastic backward scattering of protons by light nuclei [1].

In connection with this, the need has arisen of a more detailed theoretical examination of these processes.

The backward scattering of protons by deuterons has been discussed in the greatest detail in the literature. In the works of Amado [2], and of

*Numbers in the margin indicate pagination in the original foreign text.

Coleman and Overseth [3] this process was treated as deuteron stripping. Although in [3] various wave functions of the deuteron were tested, in the field of high energies large divergences from the experimental data have appeared, both for the magnitude of the cross-section and for the angular distribution and the energy dependence.

The best agreement was attained when the D-wave part of the deuteron function [4] was taken into account. However, even in this case, a significant divergence from experiments appeared when the energy was increased to 1 GeV or higher.

Another reaction mechanism was examined by Bertocci and Capella [5]. The backward scattering of the protons was regarded as the result of their multiple scattering by nucleons in the deuteron. The cross-section calculated by them turned out to be almost an order of magnitude less than the experimental value.

The most satisfactory agreement with experiments was obtained by Kerman and Kisslinger [4] (see also [6]), who returned to the exchange mechanism. Backward scattering was examined on a model of the Regge poles. The necessary /4
parameters were determined according to data concerning pion-nucleon backward scattering.

In spite of the good numerical agreement with the experiments obtained in [4] and [6], the question of the mechanism responsible for backward scattering cannot be considered closed. More detailed measurements must be carried out. Specifically, in [4] the predominant role of the exchange of resonance N* (1688) with spin 5/2 was noted, as compared with neutron exchange. Inasmuch as polarization effects may be different from case to case, it would be useful to measure them experimentally and to compare them with the calculated values.

Similar calculations do not exist for backward scattering of protons by nuclei which are more complex than deuterons. The only theoretical estimates were made by D. I. Blokhintsev and K. A. Toktarov on the basis of the so-called fluctuation model [7, 8]. In view of its qualitative character, the model determines only the dependence of the backward-scattering cross-section of protons on their energy [1].

In the language of Feynman diagrams*, the fluctuation mechanism corresponds to the process of multiple scattering of protons by the nucleons of the nucleus. As was already mentioned, there is a large divergence between the contribution of this mechanism to backward-scattering by deuterons and the experimental data, even though it describes rather well the energy behavior of the cross-section. It is difficult to calculate similar diagrams for heavier nuclei.

The present work examines a rival exchange mechanism, which holds that not the impinging proton, but rather the proton originally belonging to the nucleus, is scattered backwards.

In Figures 1 and 3 are presented the simplest exchange diagrams for $^4He$ and $^3He$ when the nucleons which are being exchanged make up a connected system, deuteron and tritium, respectively.

There are also a number of more complex exchange diagrams, which are not /5
examined in this work.

In § 2 the contribution of the polar diagram is calculated for $^4He$. All the parameters needed for calculation were obtained from data concerning elastic scattering of electrons by $^4He$ nuclei [9]. A comparison of the calculation results with the experimental data gave a good agreement. An especially interesting feature of the diagram examined here is the characteristic drop in the curve showing dependence of the cross-section on the proton energy in the vicinity of $T_p \cong 200$ Mev.

*Translator's Note: A method used in calculations involving interactions between many bodies.

In § 3, the contribution of the polar diagram to the backward scattering of protons by $^3$He nuclei is examined. Because of the absence of data about elastic scattering of electrons by $^3$He in the region of large transmitted impulses, it was not possible to calculate the magnitude of the cross-section independently. However, the data about backward scattering of protons by $^3$He make it possible, under the assumption that the contribution of the polar diagram is predominant, to make an estimate of the parameter determining the behavior of the wave function of $^3$He at short distances.

Inasmuch as the numerical agreement between the results of calculation and the experiments is insufficient to enable us to draw conclusions about the predominant role of the polar diagram, further experimental checking of its predictions is necessary. The fourth section is devoted to this question.

In the fifth section, the cross-section of the reaction d + $^4$He → $^3$He + $^3$He[(1)] is calculated. This reaction is interesting, because it makes it possible to verify the isotopic invariance of strong interactions [10, 11]. A formula has been obtained which connects the cross-section of the reaction with the cross-sections of backward scattering of protons by the $^3$He and $^4$He nuclei. It was assumed here that the polar diagrams predominate in these processes.

## 2. Backward Scattering of Protons by $^4$He /6

Let us calculate the contribution of the polar diagram shown in Figure 1 by means of non-relativistic diagram techniques [12]. All particles will be assumed to have zero spin. The amplitude corresponding to this diagram has the following form

$$A = \frac{2\,|G(\vec{q}^{\,2})|^2}{\dfrac{(\vec{k}_f - \vec{k}_i)^2}{2M_{^3H}} - \dfrac{\vec{k}_f^{\,2}}{2M_{^4He}} + \dfrac{\vec{k}_i^{\,2}}{2M_p} - \epsilon_{^4He}}. \tag{1}$$

(1) This task, suggested to one of the authors (B.Z.K.) by L. I. Lapidus, was the impetus which led to the writing of this work.

Here $\vec{k}_i$ and $\vec{k}_f$ are the momentum of the proton before scattering, and of the $^4$He nucleus after scattering in the center-of-mass system of the colliding particles;

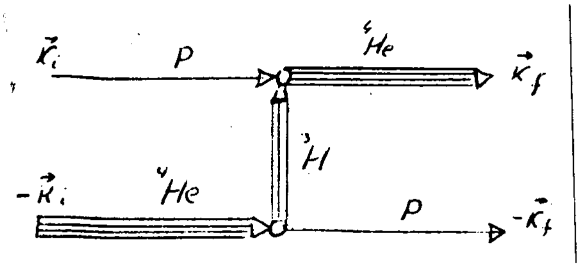


Figure 1.

$M_{4He}$, $M_{3H}$ and $M_p$ are the masses of the helium, tritium, and proton nuclei; and $\varepsilon_{4He} \approx 20$ Mev, is the bond energy of the proton and tritium in the $^4$He nucleus. The number 2 in the numerator appears because the $^4$He nucleus contains two protons. The function $G(\vec{q}^{\,2})$ corresponds to the vertex of virtual decomposition of $^4$He into proton and tritium or their fusion. It depends on the relative momentum $\vec{q}$ of the proton and tritium in their center-of-mass system:

$$\vec{q} = \vec{k}_i - \frac{1}{4}\vec{k}_f .$$

$G(\vec{q}^{\,2})$ has the following form

$$G(\vec{q}^{\,2}) = (\vec{q}^{\,2}/2\mu_{p-{}^3H} - \epsilon_{{}^4He})\,\Phi(q). \tag{2}$$

Here, $\mu_{p-{}^3H} = \dfrac{M_p \, M_{{}^3H}}{M_p + M_{{}^3H}}$ is the reduced mass of the proton and tritium; $\Phi(q)$ is the Fourier-form of the wave function of their relative movement in the $^4$He nucleus.

We may use the expression obtained by Bassel and Wilkin [13] as the wave /7 function of $^4$He in coordinate representation, determined from data concerning the scattering of electrons on $^4$He [9]:

$$|\Psi(\vec{r}_i)|^2 = N \prod_{i=1}^{4} \exp(-a^2 r_i^2)\,[1 - D\exp(-\frac{a^2}{\gamma^2} r_i^2)] . \tag{3}$$

Here $\vec{r}_i$ represents the coordinates of the $i^{th}$ nucleon in the center-of-gravity system of the nucleus; $a^2 = 0.579$ $F^{-2}$; $\gamma^2 = 0.308$; and $D = 0.858$. The expression in brackets differs significantly from one only in the region of short distances from the center of the nucleus.

Transferring to Jacobi's coordinates, we obtain an expression for the wave function of the relative motion of the proton and tritium:

$$\phi(r) = \left(\frac{3\alpha^2}{4\pi}\right)^{\frac{3}{4}} \left[\frac{(1+1/\gamma^2)^{3/2}}{(1+1/\gamma^2)^{3/2} - D}\right]^{\frac{1}{2}} \exp\left(-\frac{3}{8}\alpha^2 r^2\right) \times \left[1 - D\exp\left(-\frac{9}{16}\frac{\alpha^2}{\gamma^2} r^2\right)\right]^{\frac{1}{2}}. \tag{4}$$

Here, $\vec{r} = \vec{r}_1 - 1/3\,(\vec{r}_2 + \vec{r}_3 + \vec{r}_4)$ is the distance from the proton to the center of the tritium nucleus. In conversion to Jacobi's coordinates use was made of the circumstance that the following correlation is satisfied for the medium values of the internal coordinates of the nucleons in tritium $\vec{\rho}_j$

$$\langle \rho_j^2 \rangle \gg \frac{\gamma^2}{\alpha^2}.$$

/8

Expanding the radicand in (4) into a series and performing Fourier transformation, we obtain:

$$\Phi(q) = \left(\frac{16}{3\alpha^2}\right)^{3/4} \left[\frac{(1+1/\gamma^2)^{3/2}}{(1+1/\gamma^2)^{3/2} - D}\right]^{\frac{1}{2}} \left\{ \exp\left[-\frac{2}{3}\frac{\vec{q}^{\,2}}{\alpha^2}\right] - \frac{D}{2(1+3/2\gamma^2)^{3/2}} \exp\left[-\frac{2\vec{q}^{\,2}}{3(1+3/2\gamma^2)}\right] - \dots \right\}. \tag{5}$$

In the energy interval examined here, the series in (5) converges rapidly. Substituting (5) and (2) in (1), we calculate the differential cross-section of backward scattering of protons in the center-of-mass system by $^4$He nuclei according to the formula:

$$\frac{d\sigma}{d\Omega} = \left| \frac{\mu_{p-{}^4He}}{2\pi} A \right|^2, \tag{6}$$

where $\mu_{p-{}^4He}$ is the reduced mass of the proton and $^4$He. Figure 2 gives the results for the differential scattering cross-section for a center-of-mass angle of $\Theta = 11°$ as a function of the kinetic energy of the protons in the laboratory system. $\Theta$ is the departure angle of the $^4$He nucleus referred to the direction of the incident beam in the center-of-mass system. Its magnitude was selected according to experimental data.

The dip in the energy transfer of the cross-section at $T_p$ = 190 Mev is determined by the dip in the dependence of the electric form factor of the $^4He$ nucleus on $q^2$. There is good agreement with the experimental data in the region /9 where the impinging proton has high energy. At $T_p$ = 665 Mev (1) - (6), give

$$\frac{d\sigma}{d\Omega}\ (\Theta = 11^\circ) = 2{,}5 \cdot 10^{-31}\ cm^2/ster.$$

The corresponding experimental value is $(1.2 \pm 0.24) \cdot 10^{-31}$ $cm^2$/ster.

At $T_p$ = 150 Mev, calculations give $\frac{d\sigma}{d\Omega}(\Theta = 11^\circ) = 3 \cdot 10^{-29}$ $cm^2$/ster.

The corresponding experimental value is $(1.2 \pm 0.24) \cdot 10^{-29}$ $cm^2$/ster.

At smaller proton energies, the calculated magnitudes of the cross-section exceed the experimental data by a considerable degree.

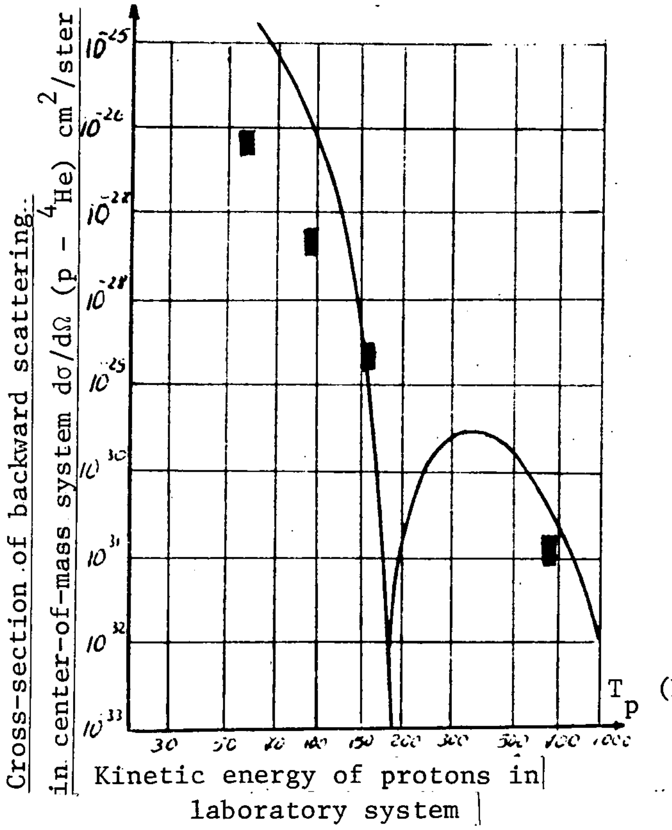


Figure 2.

However, this shortcoming is present in all polar diagrams and may possibly be connected with the necessity of introducing certain types of unitary corrections [16].

We notice that at large energies the results of calculation are very responsive to the behavior of the wave function of $^4He$ at short distances. For instance, if the wave function is selected as a simple Gaussian function, the cross-section of the process at $T_p$ = 665 Mev will drop by a factor of more than $10^{10}$.

The angular distribution has its peak in the backward direction everywhere except for a small area in Figure 2 after the dip, where a growth of the cross-section is observed. This can be explained easily by the fact that the matrix element of the process depends only on $q^2$. Therefore, the "sliding" on the curve in Figure 2 towards the bigger proton energy side corresponds to the increase in the scattering angle. /10

Figure 5 gives the angular distribution of the process at $T_p$ = 665 Mev. Experimental data are still lacking.

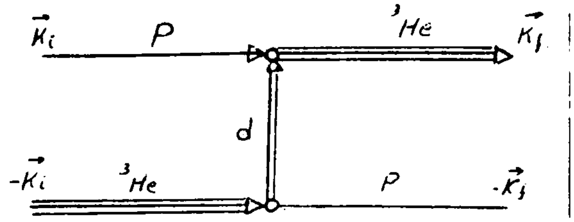


Figure 3.

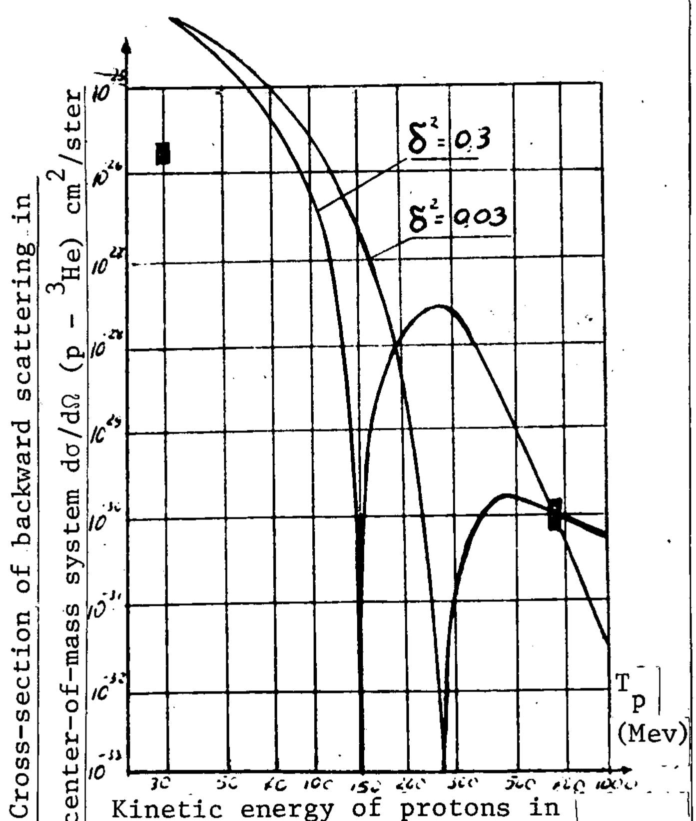


Figure 4.

The available data for protons with an energy of 150 Mev [15] give a more inclined angular dependence than is predicted by the polar diagram. However, this was only to be expected. In actual fact, different scattering mechanisms begin to predominate in the dip region, and for this reason the movement of the curve in Figure 2, and consequently also the angular distribution in the region before the dip, ought to be more inclined.

## 3. Backward Scattering of Protons by $^3$He Nuclei

In this case, the diagram in Figure 3 is calculated in an analogous manner to the calculations for $^4$He. However, it is difficult to determine the vertex parts of the diagram, since data concerning the scattering of electrons on $^3He$ are available only in the region of relatively small transmitted moments where divergences of the wave function from the Gaussian form have not yet appeared. The corresponding part of the wave function of $^3He$, which is symmetrical in its coordinates, has the following appearance [17].

$$|\Psi(\vec{r}_i)|^2 = N \prod_{i=1}^{3} \exp(-3\beta^2 r_i^2) . \tag{7}$$

Here, $\beta^2 = 0.16\ F^{-2}$; and $\vec{r}_i$ represents the coordinates of the nucleons in their center-of-gravity system. Because a "core" is present in the nucleons, the wave function may be presented in a form analogous to (4) [17]:

$$|\Psi(\vec{r}_i)|^2 = N \prod_{i=1}^{3} \exp(-3\beta^2 r_i^2)[1 - C \exp(-\frac{\beta^2}{\delta^2} r_i^2)] . \tag{8}$$

/11

If the polar diagram should predominate in the backward scattering of protons by $^3He$, there arises the possibility that the unknown parameters of the wave function (8) may be determined. Unfortunately, there are only very meager experimental data concerning backward scattering for p - $^3He$. At high energies there is only one point at $T_p$ = 665 Mev [18]. Inasmuch as the cross-section at this energy depends only weakly on the magnitude C and is very responsive to $\delta^2$, if we assume that C = 1, we obtain two possible values: $\delta^2 = 0.3$, and $\delta^2 = 0.03$. Figure 4 represents the dependence of the cross-section of backward scattering of protons in the center-of-mass system by $^3He$ on the energy of the protons in the laboratory system with these values of the parameters. The position of the dip in the energy-dependence of the cross-section depends to a great degree on the magnitude C and may differ from that given.

## 4. Possible Experimental Checks of the Mechanism

Now let us examine some variants of possible experimental checks of the role of the exchange mechanism and the polar diagrams in backward scattering of protons by light nuclei. Probably the simplest from the technical point of

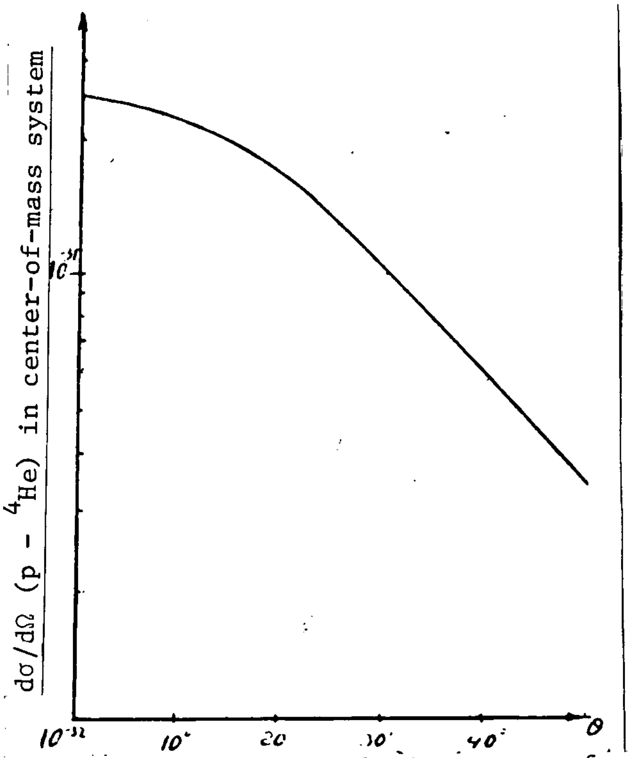


Figure 5.

view would be to compare the cross-section of backward scattering of protons by $^3H$ and $^3He$, and also to compare the cross-section of the overcharging reaction $p + {}^3H \rightarrow n + {}^3He$ at one energy. It follows from the isotopic invariance that, if the diagrams examined above predominate, the first process ought to be suppressed, and the other two processes ought to have identical cross-sections.

Of course, it would be desirable to measure the energy behavior /12
of the cross-section, especially in the region of predicted dips, as well as the angular distributions. Also interesting is the measurement of the polarization of the scattered protons or the measurement of the asymmetry in the cross-section of interaction with the polarized protons. Since such polarization or asymmetry here can arise only as the result of interference of various diagrams, should there be a strong separation of the polar diagram, it ought to be suppressed.

## 5. The Reaction $d + {}^4He \rightarrow {}^3He + {}^3H$

The simplest diagrams for this process are shown in Figure 6. The further decomposition pertains precisely to these diagrams.

Diagram 6a, illustrating proton exchange, corresponds to forward scattering of $^3He$; and Diagram 6b corresponds to forward scattering of $^3H$. For this

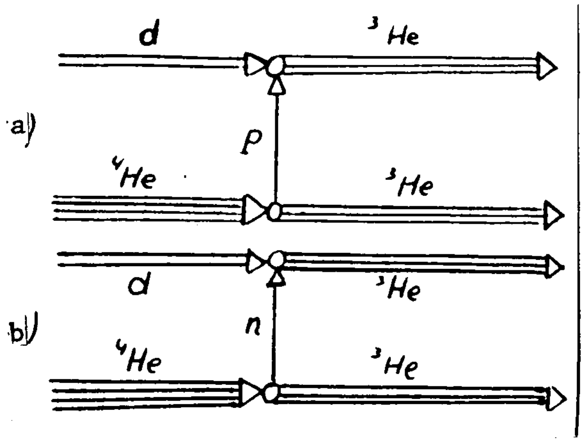


Figure 6.

reason, the interference of these diagrams may be ignored. The angular distribution of the reaction is symmetrical with respect to the substitution $\Theta$ for $\pi - \Theta$, which is in conformity with the demands of isotopic invariance.

Since the vertices in diagrams 6a and 6b were already encountered in diagrams 1 and 3, there is no difficulty in connecting with each other the differential cross-sections corresponding to these diagrams.

$$\sigma_{d+{}^4He\to{}^3He+{}^3H}(T_d) \approx \frac{10}{3}\sqrt{\sigma_{p+{}^4He\to{}^4He+p}\left(T_p = \frac{8k_d^2}{15M}\right)\cdot\sigma_{p+{}^3He\to{}^3He+p}\left(T_p = \frac{5k_d^2}{36M_p}\right)}$$

Here, $k_d = \left(M_p \cdot \frac{4T_d^2 + 16T_d M_p}{9M_p + 2T_d}\right)^{\frac{1}{2}}$ is the momentum of the deuteron in the center-of-mass system, and $T_p$ and $T_d$ represent the kinetic energies of the impinging proton and deuteron in the laboratory system. All the cross-sections pertain to the center-of-mass system at $\Theta = 11°$.

The results obtained above for backward scattering of protons may /13
possibly also be useful in other direct nuclear reactions, for example in the reactions ($^3He$, d), ($^3He$, p), or ($^4He$, $^3H$), ($^4He$, p) in various nuclei.

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